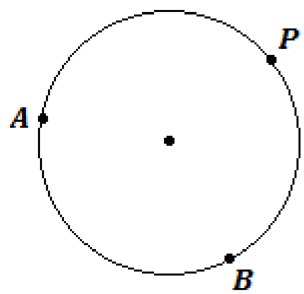
Central Angle Theorem

The *Central Angle Theorem* states:

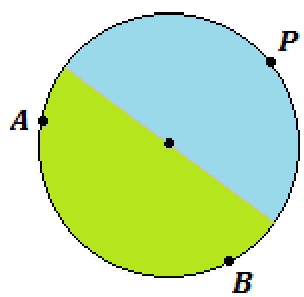
The central angle drawn from any two points on a circle is twice as large as any inscribed angle drawn from those two points.

Explanation and proof:

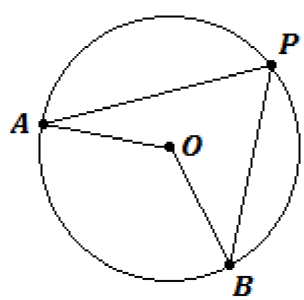
Draw a circle, and label three points on it ***A***, ***B*** and ***P***. Also label the center of the circle ***O***.



The points can be anywhere you want, as long as ***P*** is not one the same side as ***A*** and ***B***.

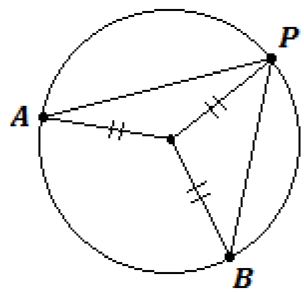


Now construct the following line segments: $\overline{OA}$, $\overline{OB}$, $\overline{PA}$, and $\overline{PB}$

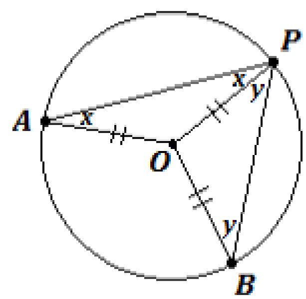


The angle $\angle AOB$ is a *central angle*, and the angle $\angle APB$ is an *inscribed angle*.

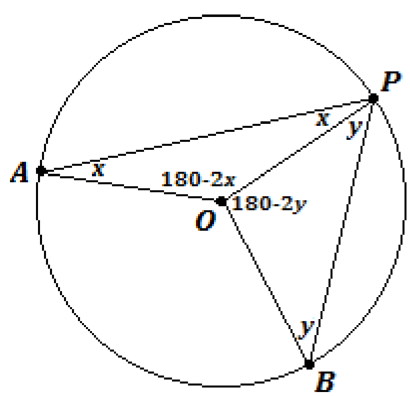
Now construct the line segment $\overline{OP}$, and note that all radii have the same length.



Since $\triangle AOP$ and $\triangle ABP$ have two equal line segments, they are isosceles, and their base angles must be congruent. Label the congruent angles *x* and *y*.



The interior angles of a triangle must sum to $180°$, so angles $\angle AOP$ and $\angle BOP$ must measure $180 - 2x$ and $180 - 2y$ respectively.



The sum of the angles around the origin must add up to $360°$. Therefore,

$$\angle AOB + 180 - 2x + 180 - 2y = 360$$

Subtracting 360 from both sides, we have

$$\angle AOB = 2x + 2y$$

Meanwhile, we can see from the above diagram that

$$\angle APB = x + y$$

So we must conclude that $\angle AOB$ has twice the measure of $\angle APB$.

$$\boxed{\angle AOB = 2 \cdot \angle APB}$$

Q.E.D.

Note: this theorem only applies if ***P*** lies on the longer of the two arcs (the *major arc*) connecting ***A*** and ***B***. If ***P*** lies on the other arc (the *major arc*), then the equation becomes

$$\angle AOB = 360 - 2 \cdot \angle APB$$

This is why I earlier insisted that ***P*** should be on the opposite side of ***A*** and ***B***.